Student Name: ______________________________

Student Number: ____________________________

GE 213.3 - Mechanics of Materials

FINAL EXAMINATION

April 19, 2004

Quest. 1:_____
Quest. 2:_____
Quest. 3:_____
Quest. 4:_____
Quest. 5:_____
Quest. 6:_____

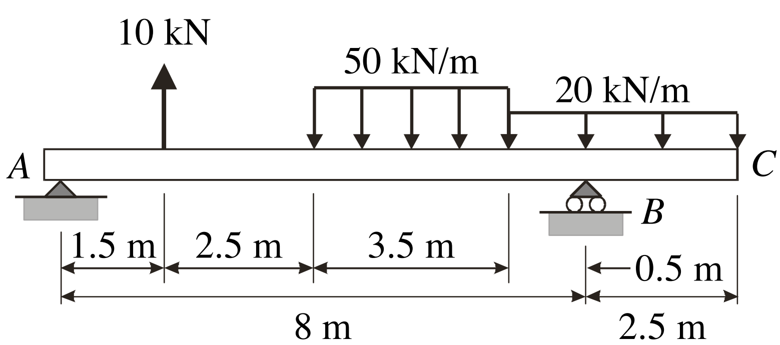
Professor: B. Sparling

Time Allowed: 3 Hours

Notes:
- Closed book examination; Calculators may be used
- The value of each question is provided along the left margin
- Supplemental material is provided at the end of the exam (formulas)
- Show **all** your work, including all formulas, calculations and units
- Write your work in the space provided on the examination sheet. (The backs of the examination sheets may also be used if required)

MARKS

15 **QUESTION 1:** Draw the shear and bending moment diagrams for Beam *ABC* in the locations indicated below.



Shear Diagram

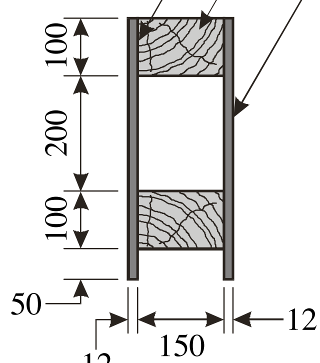
Moment Diagram

20 **QUESTION 2:** A 1.2 m long timber box beam is simply supported at both ends and is constructed by gluing two 12 mm thick x 450 mm high plywood sheets to two 100 mm x 150 mm boards, as shown in Section A-A below. At the location of Section A-A, determine the following values:

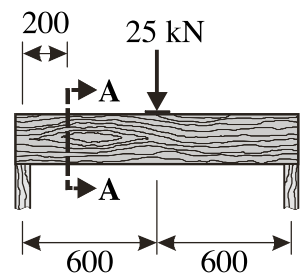
a) The maximum compressive normal stress;

b) The shear stress along the glue joints at the top of the beam; and

c) The maximum shear stress in the plywood.



Section A-A



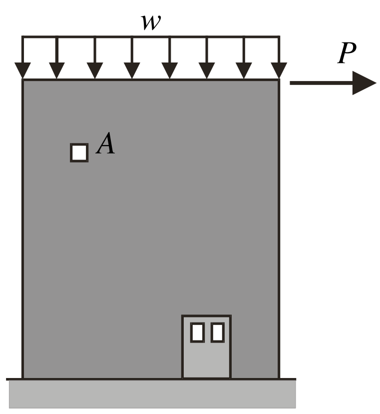
Side view of beam

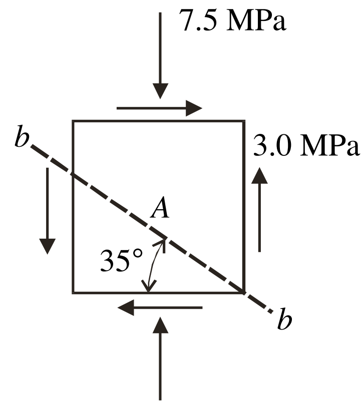
16 **QUESTION 3:** At Point *A* on a concrete wall, the state of stress due to the indicated loading condition is shown below on the right. Using the Mohr's Circle method, determine the following stresses and illustrate the orientation and direction of the stresses on a sketch:

a) The principal normal stresses; and

b) The normal and shear stresses on a plane defined by Line *b-b*.

Note: The stress transformation equations, rather than Mohr's circle, can be used; however, there will be a **5 mark penalty** assigned for using this method.



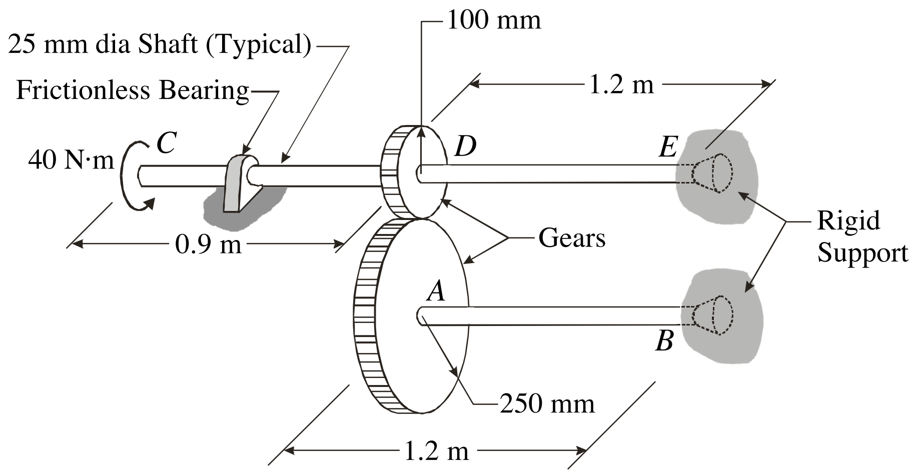


State of Stress at Point A

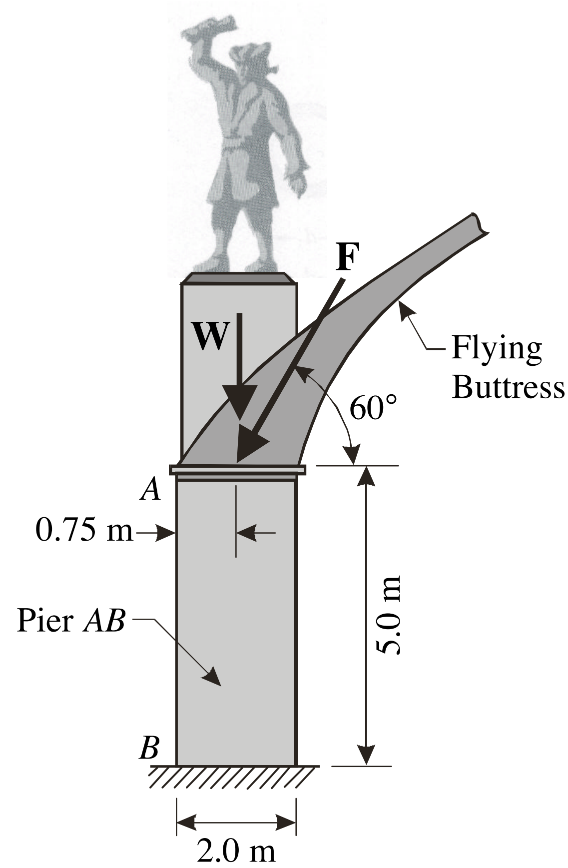
18 **QUESTION 4:** Solid shafts *AB* and *CDE* are connected by the two gears shown below, and are fixed to rigid supports at Ends *B* and *E* such that there is no rotation at either point. If both shafts are the same size (25 mm diameter) and are made of steel with a Modulus of Rigidity of G = 77,000 MPa, determine:

a) The contact force between the two gears; and

b) The angle of twist at Point *C*.

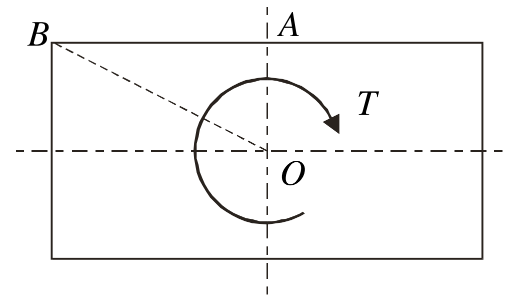


16 **<u>QUESTION 5:</u>** A flying buttress supporting the roof of a Gothic church exerts an inclined reaction force **F** = 25 kN, acting at an angle of 60° from the horizontal, onto the top of vertical Pier *AB*. The vertical Pier *AB* has a rectangular cross-section with a width of 3.0 m into the page. The stone used to build the Pier *AB* weighs 2,600 N/m^3. Determine the minimum weight **W** of the statue and its pedestal (i.e. the weight of everything above Pier *AB*) required to avoid any tensile normal stress at the base of Pier *AB*. Assume that applied forces **W** and **F** act at the centre of the top end of the pier.

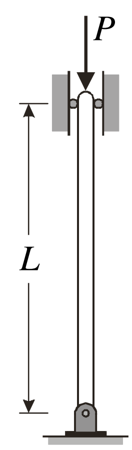


QUESTION 6: Provide brief answers to the following questions – answers in point form are acceptable. Diagrams should be used to supplement your responses where appropriate.

5 **a)** A shaft with the rectangular cross-section shown below is subjected to torque T about its longitudinal axis. Sketch the approximate shape of the torsional shear stress distribution along Lines OA and OB (Point O is on the longitudinal axis) – **do not** do any calculations. Also indicate the location of the maximum shear stress within the shaft.

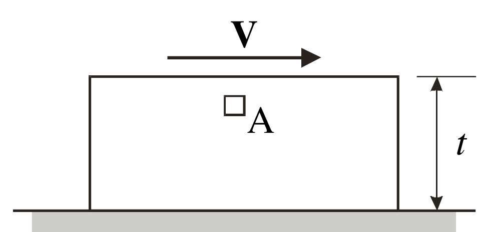


6 **b)** In a qualitative sense (no formulas), describe the axial load (P) versus lateral (sideways) displacement behaviour of the slender, ideal elastic column shown below as the axial load is increased from a value of zero until failure occurs. Clearly identify and describe the significant phases in the response.



Initial Position

4 **c)** A block of material with a small thickness t is subjected to a horizontal force **V** along its top surface. Show a sketch illustrating the shear deformation that will occur on the small element located at Point A, as shown. Using this sketch, clearly demonstrate the definition of shear strain, γ , in that element.



- **Normal Stress:** $\sigma_{avg} = \dfrac{P}{A}$ $\quad P = \int_A \sigma \, dA$ • **Bearing Stress:** $\sigma_b = \dfrac{P}{t\,d}$
- **Direct Shear:** $\tau_{avg} = \dfrac{V}{A}$ (Single) or $\tau_{avg} = \dfrac{V}{2A}$ (Double) • **Hooke's Law:** $\sigma = E\,\varepsilon$
- **Allowable Stress:** $F.S. = \dfrac{P_U}{P_D}$ or $F.S. = \dfrac{\sigma_U}{\sigma_D}$; $\sigma_{all} = \dfrac{\sigma_U}{F.S.}$ $\quad P_{all} = \sigma_{all}\, A$ $\quad A_{req} = \dfrac{P_D}{\sigma_{all}}$
- **Stresses on Oblique Planes:** $\sigma_\theta = \dfrac{P\cos\theta}{A_o/\cos\theta} = \dfrac{P}{A_o}\cos^2\theta$; $\tau_\theta = \dfrac{P\sin\theta}{A_o/\cos\theta} = \dfrac{P}{A_o}\sin\theta\cos\theta$
- **Average Normal Strain:** $\varepsilon = \dfrac{\delta}{L_o} = \dfrac{L^* - L}{L}$ • **Poisson's Ratio:** $\varepsilon_y = \varepsilon_z = -\nu\,\varepsilon_x$
- **Axial Deformations:** $\delta = \dfrac{P\,L_o}{A_o\,E}$; $\quad \delta_{tot} = \sum_i \dfrac{P_i\,L_i}{A_i\,E_i}$; $\quad \delta = \int_0^L \dfrac{P(x)}{A(x)\,E(x)}\,dx$
- **General Hooke's Law:** $\varepsilon_x = \dfrac{\sigma_x}{E} - \nu\dfrac{\sigma_y}{E} - \nu\dfrac{\sigma_z}{E}$; $\varepsilon_y = -\nu\dfrac{\sigma_x}{E} + \dfrac{\sigma_y}{E} - \nu\dfrac{\sigma_z}{E}$; $\varepsilon_z = -\nu\dfrac{\sigma_x}{E} - \nu\dfrac{\sigma_y}{E} + \dfrac{\sigma_z}{E}$
- **Shearing Strain & Stress:** $\theta^* = \dfrac{\pi}{2} - \gamma_{xy}$; $\quad \gamma_{xy} = \dfrac{\tau_{xy}}{G}$; $\gamma_{yz} = \dfrac{\tau_{yz}}{G}$; $\gamma_{zx} = \dfrac{\tau_{zx}}{G}$; $\quad G = \dfrac{E}{2(1+\nu)}$
- **Thermal Deformations:** $\delta_T = \alpha\,(\Delta T)\,L_o$; $\varepsilon_T = \dfrac{\delta_T}{L_o}$ • **Resultant Torque:** $T = \int_A \rho\,\tau\,dA$
- **Torsional Strains:** $\gamma = \dfrac{\rho\,\phi}{L}$; $\quad \gamma_{max} = \dfrac{c\,\phi}{L}$; $\quad \gamma = \left(\dfrac{\rho}{c}\right)\gamma_{max}$
- **Torsional Stresses:** $\tau = \left(\dfrac{\rho}{c}\right)\tau_{max}$ $\quad \tau_{max} = \dfrac{T\,c}{J}$ $\quad \tau = \dfrac{T\,\rho}{J}$ $\quad J = \int_A \rho^2\,dA = \dfrac{\pi}{2}c^4$
- **Torsional Angle of Twist:** $\phi = \dfrac{T\,L}{J\,G}$ • **Torsion - Gear Compatibility:** $\phi_1\,\rho_1 = \phi_1\,\rho_2$
- **Pure Bending - Normal Strain:** $\varepsilon_x = -\dfrac{y}{\rho}$ $\quad \varepsilon_{max} = c/\rho$ $\quad \varepsilon_x = -\dfrac{y}{c}\varepsilon_m$
- **Pure Bending - Normal Stress:** $\sigma_x = -\dfrac{y}{c}\sigma_m$ $\quad \sigma_x(y) = -\dfrac{M\,y}{I}$ $\quad \sigma_{max} = \dfrac{M\,c}{I}$
- **Section Properties:** $I = \int_A y^2\,dA$; $\; I = \sum_i \left(I_i + A_i\,d_i^2\right)$; Centroid: $\int_A y\,dA = 0$; $\; \bar{y}\,A = \sum_i y_i\,A_i$
- **Biaxial Bending:** $\sigma_x = -\dfrac{M_z\,y}{I_z} + \dfrac{M_y\,z}{I_y}$; $\; \tan\phi = \dfrac{I_z}{I_y}\tan\theta$; $M_z = M\cos\theta$; $M_y = M\sin\theta$
- **Eccentric Axial Loading:** $\sigma_x = \dfrac{P}{A} - \dfrac{M\,y}{I}$; • **Shear Flow:** $q = V\,Q/I$
- **Flexural Shear Stress:** $\tau_{ave} = \dfrac{V\,Q}{I\,t}$; $\; Q = \int_A y\,dA = A\,\bar{y}$ • **Discrete Fasteners:** $F_N = q \times s$
- **Plane Stress Transformations:** $\sigma_{x'} = \dfrac{\sigma_x + \sigma_y}{2} + \dfrac{\sigma_x - \sigma_y}{2}\cos(2\theta) + \tau_{xy}\sin(2\theta)$

$\sigma_{y'} = \dfrac{\sigma_x + \sigma_y}{2} - \dfrac{\sigma_x - \sigma_y}{2}\cos(2\theta) - \tau_{xy}\sin(2\theta)$; $\quad \tau_{x'y'} = -\dfrac{\sigma_x - \sigma_y}{2}\sin(2\theta) + \tau_{xy}\cos(2\theta)$

- **Principal Normal Stress:** $\sigma_{p1,p2} = \dfrac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$; $\tan(2\theta_p) = \dfrac{\tau_{xy}}{(\sigma_x - \sigma_y)/2}$
- **Maximum Shear Stress:** $\tau_{max} = \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$; $\sigma_{avg} = \dfrac{\sigma_x + \sigma_y}{2}$; $\tan(2\theta_s) = -\dfrac{(\sigma_x - \sigma_y)/2}{\tau_{xy}}$
- **Mohr's Circle:** $\sigma_{avg} = \dfrac{\sigma_x + \sigma_y}{2}$; $\; R = \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$; $\sigma_{p1,p2} = \sigma_{avg} \pm R$; $\tau_{max} = R$